Question 6

Show that

$$\frac{\sum x_i^2 \sum y_i - \sum x_i \sum x_i y_i}{n \sum (x_i - \bar{x})^2} = \hat{Y} - \hat{\beta}\bar{x}$$

$$\beta = (x'x)^{-1} x'y$$

$$\hat{Y} - \hat{\beta}\bar{x} \qquad \hat{\beta} = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n} (x_i - \bar{x})^2}$$

$$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$$

$$= (\bar{y} - \beta_1 \bar{x}) + \hat{\beta}_1 x_i$$

$$= \bar{y} + \hat{\beta}_1 (x_i - \bar{x})$$

$$= \bar{y} + \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n} (x_i - \bar{x})^2} \left(x_i - \bar{x} \right)$$

$$= \frac{\sum y_i}{n} + \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n} (x_i - \bar{x})^2} \cdot \left(x_i - \frac{\sum x_i}{n} \right)$$

$$= \left(\frac{\sum y_i}{n} - \left(\frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n} (x_i - \bar{x})^2} \right) \bar{x} \right) +$$

$$x_i \left(\frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n} (x_i - \bar{x})^2} \right)$$

$$= \frac{\sum x_i^2 \sum y_i - \sum x_i \sum x_i y_i}{n \left(\sum (x_i - \bar{x})^2 \right)}$$